Name and Surname : Solns

Grade/Class : 12/……….   **Mathematics Teacher** : ……………………………

150

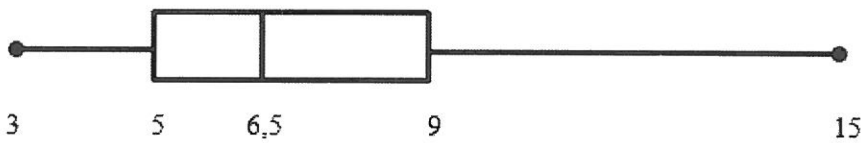
ANSWER BOOKLET

June Paper 2

2019

QUESTION 1

1.1.

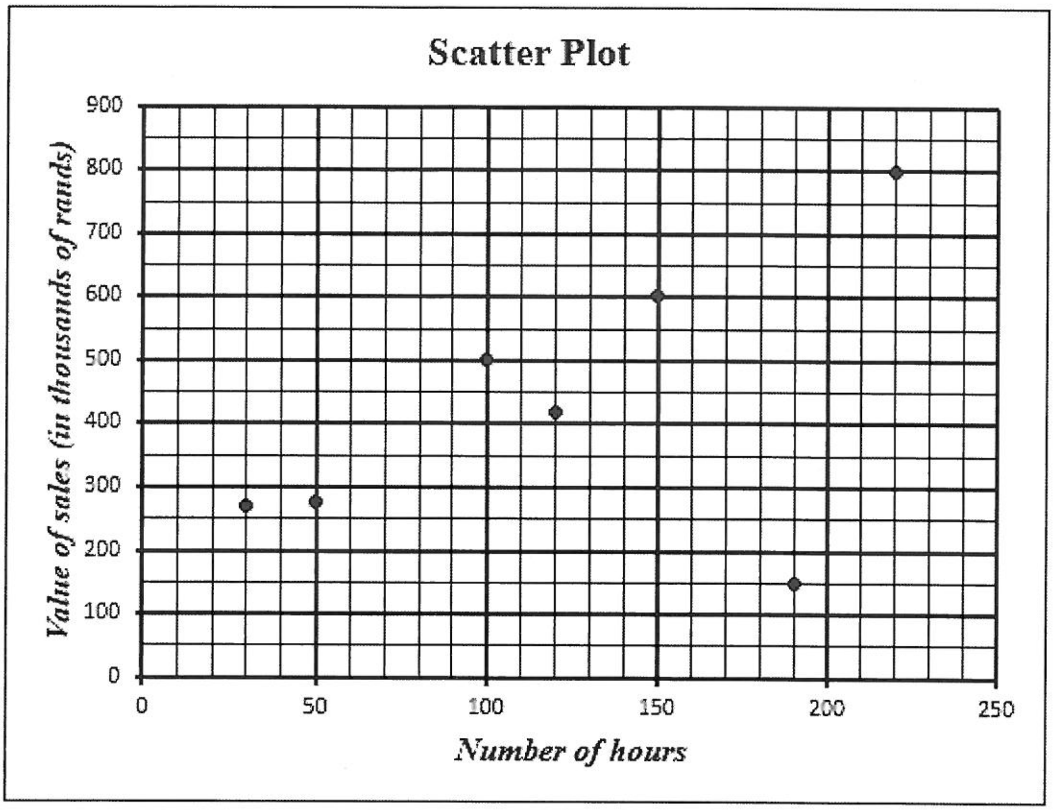


1.1.1.	Positively skewed OR skewed to the right	1
1.1.2.	25 %	1
1.1.3.	$S\text{-}IQR = \frac{9-5}{2}$	
	$= 2$	1

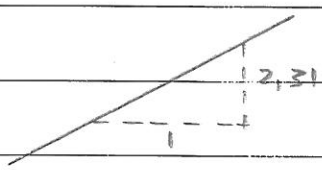
1.2.

88,862 213,358

Number of hours	30	50	100	120	150	190	220	240	260
Value of sales (in thousands of rands)	270	275	500	420	602	150	800	850	820



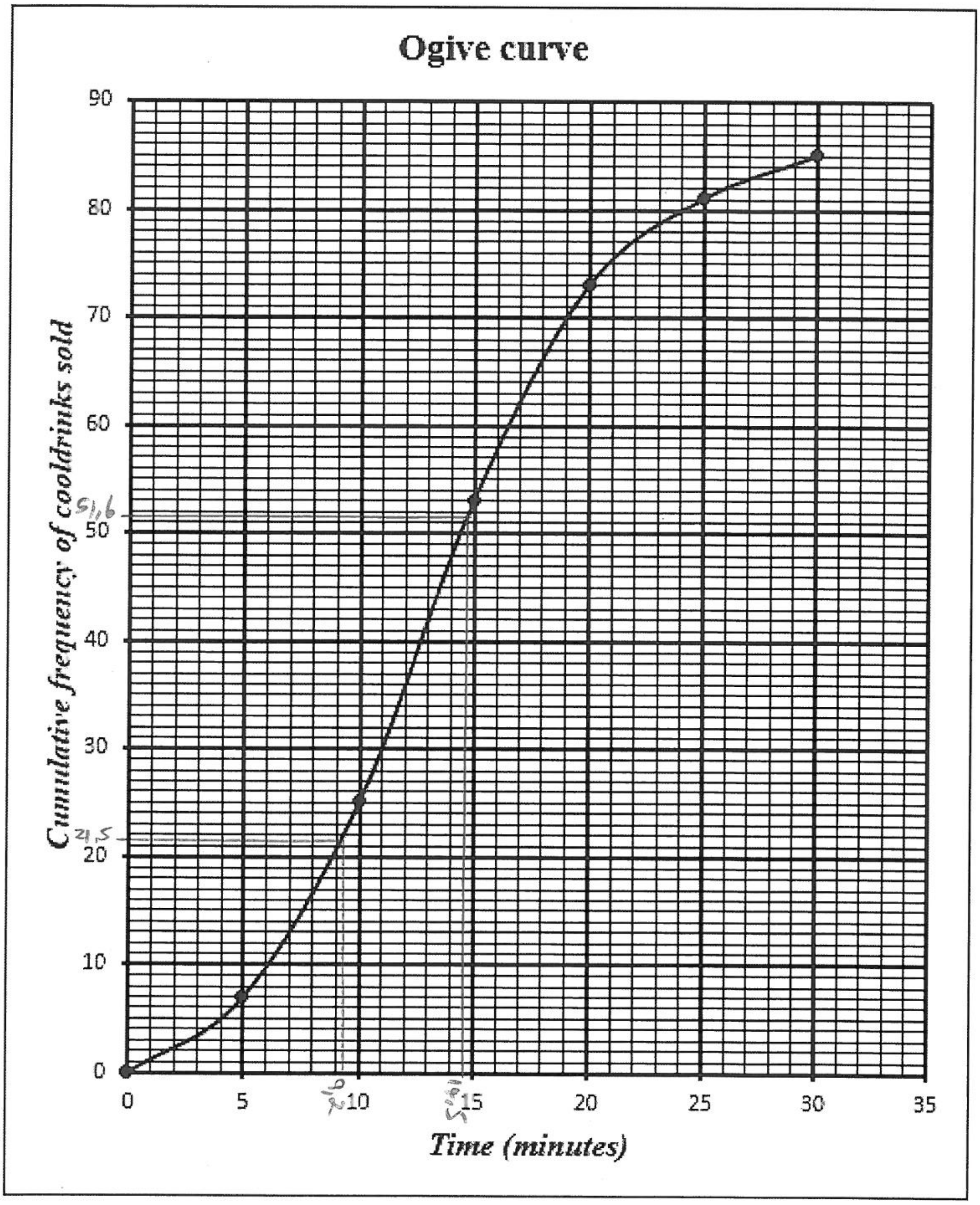
1.2.1.	(190 ; 150) ✓	1
1.2.2.	As the number of hours increases	
	the value of sales increases. ✓	1
1.2.3.	A = 171,37 ✓ B = 2,31 ✓	
	∴ $y = 171,37 + 2,31x$ ✓	3

1.2.4.	$y = 171,37 + 2,31(80)$ ✓	
	$= 356,17$	
	$\therefore$ R 356 000 ✓ Accept 356	2
1.2.5.	$r = 0,73$ ✓	
	$\therefore$ there is a strong ✓ positive	
	linear relationship	2
1.2.6.	$B = m$	
	$= 2,31$	
		
	B	
	$\therefore$ 2,31 ✓ × 1000	
	= R 2310 ✓ Accept 2312	2
1.2.7.	(a) $\bar{x} = 151,11$ hrs ✓	1
	(b) $\sigma_x = 77,81$ hrs ✓	1

1.2.7	(c) $\bar{x} \pm 0{,}8\sigma$	
	$= 151{,}11 \pm 0{,}8 \cdot 77{,}81$	
	$= 88{,}862$ or $213{,}358$ ✓ both	
	$\therefore$ 5 clients ✓ ans only ½	2

QUESTION 2

2.

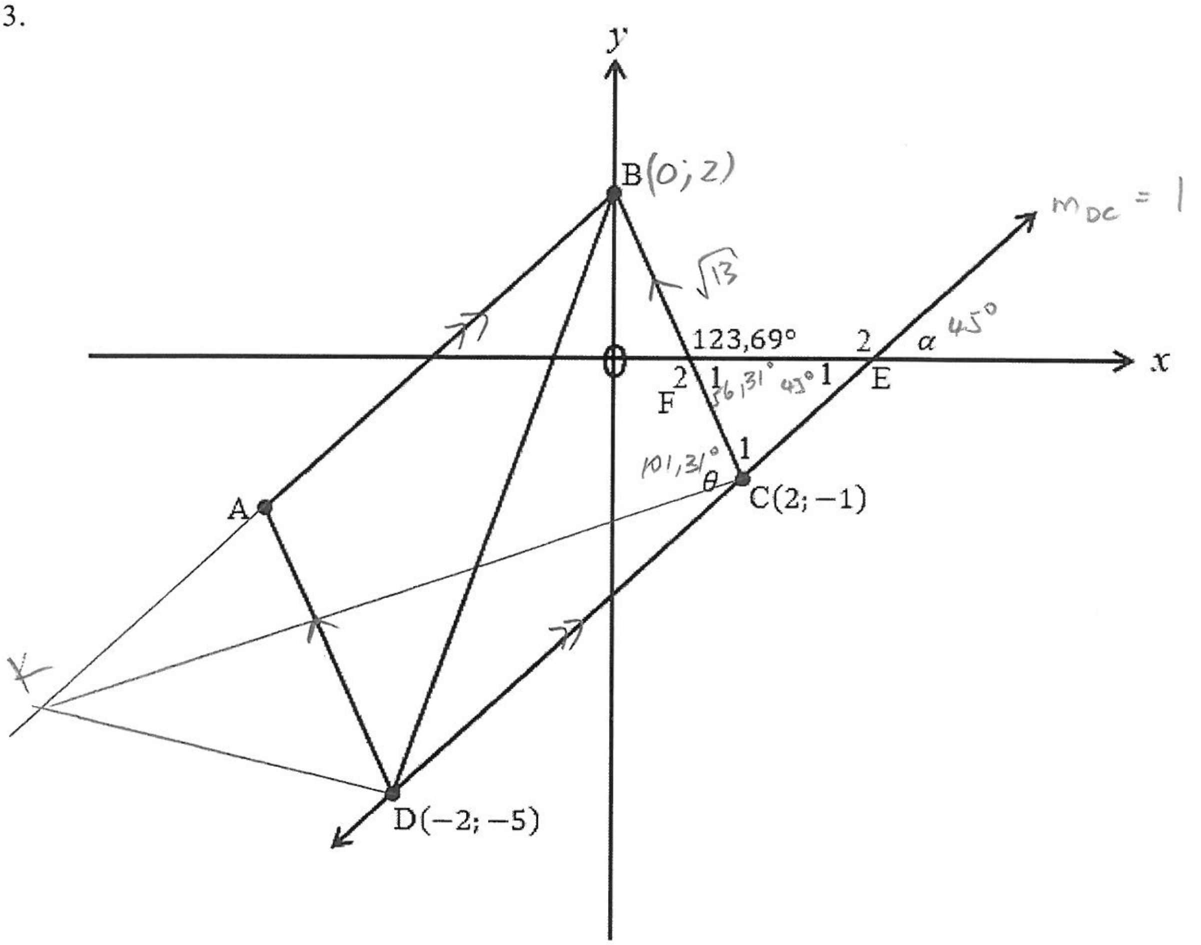


2.1	85 cooldrinks ✓	1

2.2.	$10 < t \leq 15$ min 10 – 15 $10 \leq t < 15$	1
2.3.	$t_{30} = 85$ $t_{20} = 73$	
	$\therefore$ last 10 min	
	$= 85 - 73$	
	$= 12$ cooldrinks	1
2.4.	$T_1 ; \ldots ; T_{85}$	
	$M = T_{\frac{1}{2}(1+85)} = T_{43}$ items	
2.4.1.	$T_1 ; \ldots ; T_{42}$	
	$\therefore Q_1 = T_{\frac{1}{2}(1+42)}$	
	$= T_{21,5}$ items	
	$= 9,2$ min 8 – 10	2
	ans only ½	
2.4.2.	$P_{60} = T_{\frac{60}{100}(1+85)}$	
	$= T_{51,6}$ items	
	$= 14,5$ min 13 – 15	2
	ans only ½	

QUESTION 3

3.



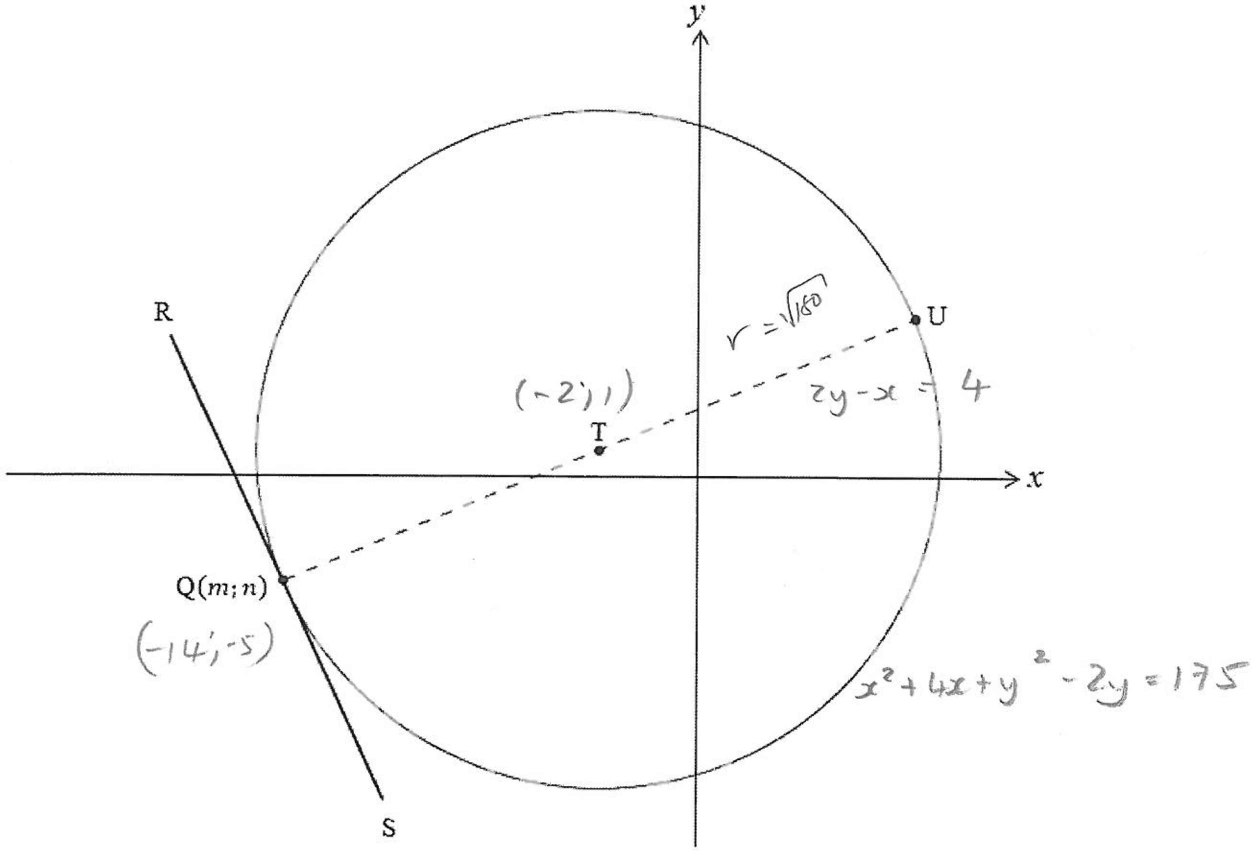
3.1.	$m_{DC} = \frac{-5-(-1)}{-2-(2)}$ ✓ D(−2;−5) C(2;−1)	
	$= 1$ ✓	2
3.2.	$\alpha = 45°$ ✓	1
3.3.	$\hat{E}_1 = 45°$ ✓S vert opp ∠'s =	
	$\hat{F}_1 = 56,31°$ ✓S ∠'s str line = 180°	

	$\theta = 101,31°$ ✓S ext ^ Δ	3
3.4.1	$m_{BC} = \tan 123,69°$ B(0;y) C(2;-1)	
LHS	✓ $\frac{y-(-1)}{0-2} = -1,5...$ ✓ RHS	
	$y+1 = 3$	
	$y = 2$	
	$\therefore B(0;2)$	4
	✓ ✓	
3.4.2.	$C(2;-1) \xrightarrow[\leftarrow 4]{\downarrow 4} D(-2;-5)$	
	$B(0;2) \xrightarrow[\leftarrow 4]{\downarrow 4} A(-4;-2)$	2
	✓ ✓	
3.5.	CD C(2;-1) D(-2;-5)	
	$= \sqrt{(-5-(-1))^2 + (-2-(2))^2}$ ✓	
	$= \sqrt{32}$	
	$= \sqrt{16.2}$ ✓ must show	
	$= \sqrt{16}.\sqrt{2}$	
	$= 4\sqrt{2}$ ✓	3

3.6.	area ΔCDK	
	$= \text{area } \Delta BDC$ ✓R same base, DC	
	✓R same height, $KB \parallel CD$	
	$= \frac{1}{2} \cdot CD \cdot BC \cdot \sin\theta$	
	$= \frac{1}{2}(4\sqrt{2})(\sqrt{13}) \sin 101,31°$ ✓	
	$= 10 \text{ u}^2$ ✓	4

QUESTION 4

4.



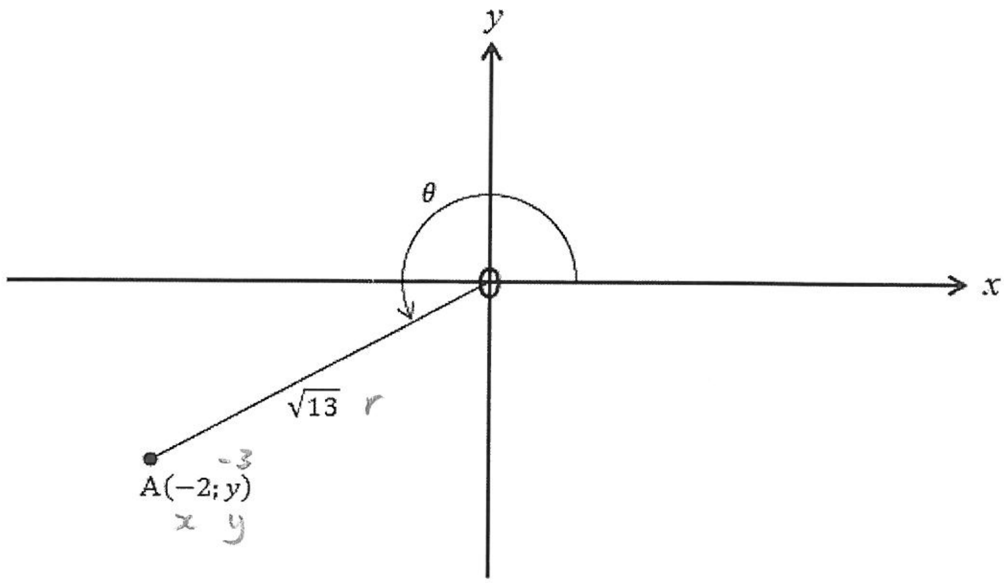
4.1	$x^2+4x+y^2-2y=175$ $\qquad$ $2y-x=4$	
	$2y-4=x$ ✓	
	$(2y-4)^2+4(2y-4)+y^2-2y=175$ ✓	
	$4y^2-16y+16+8y-16+y^2-2y-175=0$	
	$5y^2-10y-175=0$	
	$\div 5$: $\quad y^2-2y-35=0$ ✓	
	$(y-7)(y+5)=0$ ✓	
	$\therefore \quad y=\cancel{7}$ or -5	
	reject $m<0$	

	$\therefore \quad x = 2(-5) - 4$ ✓	
	$= -14$	
	$\therefore \quad m = -14$ and $n = -5$	
	$\therefore \quad Q(-14; -5)$	5
4.2.	rad $\quad 2y - x = 4$	
	$2y = x + 4$	
	$y = \frac{1}{2}x + 2$	
	$\therefore \quad m_{rad} = \frac{1}{2}$ ✓	
	$\therefore \quad m_{tan} = -2$ ✓S ✓R tan $\perp$ rad	
	$\therefore \quad y = -2x + c$	
	sub $Q(-14; -5)$	
	$-5 = -2(-14) + c$ ✓	
	$-33 = c$	
	$\therefore \quad y = -2x - 33$ ✓	5
4.3.1.	$x^2 + 4x + (+2)^2 + y^2 - 2y + (-1)^2 = 175 + 4 + 1$	
	$(x + 2)^2 + (y - 1)^2 = 180$	
	✓ ✓ ✓	3

4.3.2.	$T(-2;1)$	2
4.3.3.	$-2 = \frac{x_u + (-14)}{2}$ $\quad 1 = \frac{y_u + (-5)}{2}$	
	$10 = x_u$ $\quad 7 = y_u$	
	$\therefore\ U(10;7)$ $\quad QT = TU$, radii	2
4.3.4.	$T(-2;1)$ $\quad r = \sqrt{180}$	
	$9 \rightarrow$ $\quad 10 \downarrow$ $\quad r = \frac{\sqrt{180}}{2}$	
	$(-2+9;\ 1-10)$	
	$(7;-9)$ $\quad r = 3\sqrt{5}$	
	$\therefore\ (x-7)^2 + (y+9)^2 = (3\sqrt{5})^2$	
	$(x-7)^2 + (y+9)^2 = 45$	3

QUESTION 5

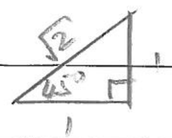
5.



5.1.1.	(a) $(-2)^2 + y^2 = (\sqrt{13})^2$ Pythag	
	$y^2 = 9$	
	$y = \pm 3$	
	$\therefore y = -3$ ✓	1
	(b) $\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$	
	$\frac{x}{r}$ $-\frac{2}{\sqrt{13}}$ ✓ $= 1 - 2\sin^2\left(\frac{\theta}{2}\right)$	
	$2\sin^2\left(\frac{\theta}{2}\right) = 1 + \frac{2}{\sqrt{13}}$	
	$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2} + \frac{1}{\sqrt{13}}$ ✓	
	$= \frac{\sqrt{13} + 2}{2\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$ ✓	
	$= \frac{13 + 2\sqrt{13}}{2.13}$	
	$= \frac{13 + \sqrt{4.13}}{26}$	

	$= \frac{13+\sqrt{52}}{26}$ ✓	4
5.1.2.	$\cos\theta = -\frac{2}{\sqrt{13}}$ ✓	
	ref $\hat{}$ = 56,30...°	
	Cos - ve	
	II : reject	
	III : $\theta = 236,31°$ ✓	2
5.2.1.	$-2\sin 4x = \sqrt{12}.\cos 4x$	
	$A = 4x$	
	$-2\sin A = \sqrt{12}\cos A$	
	$\div \cos A$: $\tan A = -\frac{\sqrt{12}}{2}$ ✓	
	$= -\sqrt{3}$	
	ref $\hat{}$ = 60°	
	tan - ve ✓ $(k \in Z)$	
	II : $A = 120° + k.180°$	
	$4x = 120° + k\,180°$	
	$x = 30° + k.45°$	4
	✓ ✓	

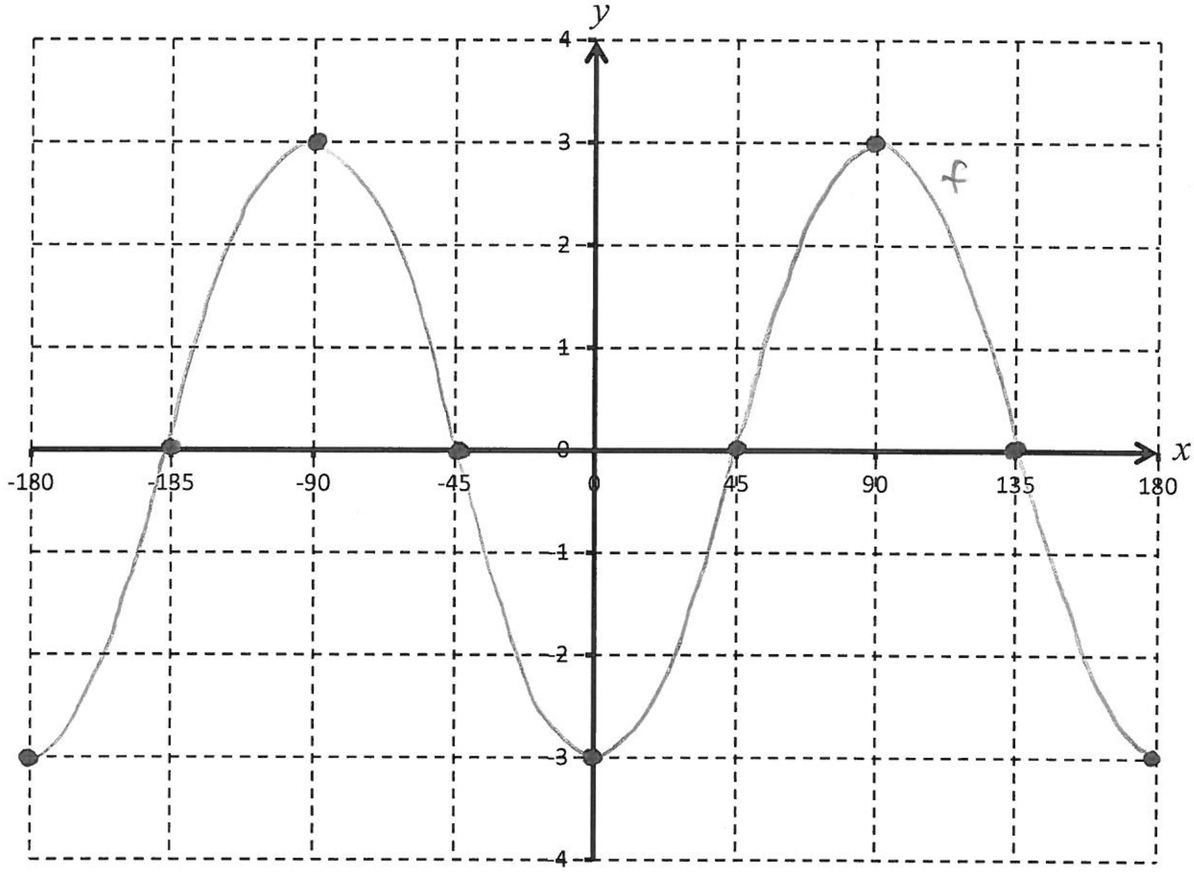
5.2.2.	$\sin 2x + \cos(x+30°) = 0$	
	$A = 2x \qquad B = x + 30°$	
	$\sin A = -\cos B \qquad (k \in \mathbb{Z})$	
	$\sin(270° - B) \qquad \sin(270° + B)$	
	$\underline{\text{III}} \qquad \underline{\text{IV}}$	
	$\sin A = \sin(270° - B)$ or $\sin A = \sin(270° + B)$	
	$A = 270° - B + k.360° \qquad A = 270° + B + k360°$	
	$2x = 270° - (x+30°) + k\,360° \qquad 2x = 270° + x + 30° + k\,360°$	
	$= 270° - x - 30° + k.360° \qquad x = 300° + k.360°$	
	$3x = 240° + k.360°$	
	$x = 80° + k.120°$	4
5.3.	$(\sin(x - 1980°) - \cos(-x))^2$	
	• $\sin(x - 1980°) = \sin(x + 180°)$	
	$= \sin(180° + x)$	
	$= -\sin x$	
	• $\cos(-x) = +\cos x$	
	$(-\sin x - \cos x)^2$	
	$= \sin^2 x + 2\sin x\cos x + \cos^2 x$	
	$= 1 + \sin 2x$	5

5.4.1.	$\sin 50^\circ = \sin(90^\circ - 40^\circ)$	
	$= \cos 40^\circ$ ✓	1
5.4.2.	No of terms $= 50 - 40 + 1$	
	$= 11$ ✓	1
5.4.3.	$\sum_{x=40^\circ}^{50^\circ} \sin^2 x$	
	$= \sin^2 40^\circ + \sin^2 41^\circ + \dots \sin^2 44^\circ$	
	$+ \sin^2 45^\circ$	
	$+ \sin^2 46^\circ + \dots + \sin^2 49^\circ + \sin^2 50^\circ$ ✓ expand	
	$= \sin^2 40^\circ + \sin^2 41^\circ + \dots + \sin^2 44^\circ$	
	$+ (\sin 45^\circ)^2$	
	$+ \cos^2 44^\circ + \dots + \cos^2 41^\circ + \cos^2 40^\circ$ ✓ co-ratios	
	$= \sin^2 40^\circ + \cos^2 40^\circ + \dots + \sin^2 44^\circ + \cos^2 44^\circ$	
	$+ \left(\frac{1}{\sqrt{2}}\right)^2$ ✓	
	$= \underbrace{1 + 1 + 1 + 1 + 1}_{} + \frac{1}{2}$ $\quad \frac{10}{2} = 5$	
	$= 5 + \frac{1}{2}$	
	$= \frac{11}{2}$ ✓	
		4

xint ✓
tp's ✓
yint + shape ✓

QUESTION 6

6.1. $f(x) = 6\sin^2 x - 3$



3

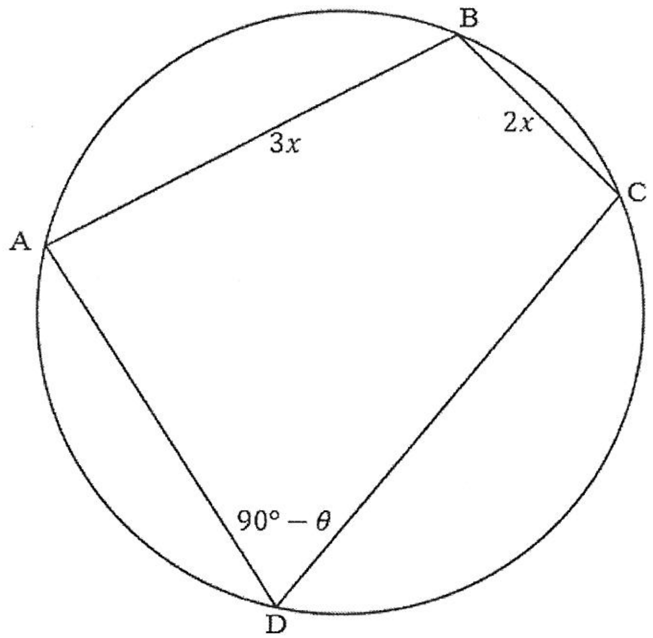
6.2.1.	Period = $180°$ ✓	1
6.2.2.	Amplitude = 3 ✓	1
6.3.1.	$6\sin^2 x - 3 = -1$	
	$\sin^2 x = \frac{1}{3}$	
	$\sin x = \pm\sqrt{\frac{1}{3}}$ ✓	

	ref $\hat{}$ = $35,26...^\circ$	
	sin ± in $(k \in \mathbb{Z})$	
	I : $x = 35,26^\circ + k.360^\circ$ ✓	
	II : $x = 144,74^\circ + k.360^\circ$	
	III : $x = 215,26^\circ + k.360^\circ$ ✓	
	IV : $x = 324,74^\circ + k\,360^\circ$	3
6.3.2.	$6\sin^2 x - 3 = -1$ when	
	$x = -144,74^\circ; -35,26^\circ; 35,26^\circ$ or $144,74^\circ$	
	$\therefore$ $f(x) < -1$ when	
	$x \in [-180^\circ; -144,74^\circ)$ ✓ or $(-35,26^\circ; 35,26^\circ)$ ✓	
	or $(144,74^\circ; 180^\circ]$ ✓	3
6.4.	$f(x) = 3(2\sin^2 x - 1)$	
	$= -3(1 - 2\sin^2 x)$	
	$= -3\cos 2x$ ✓	
	$g(x) = -3\cos(2x + 70^\circ) + 2$	
	$= -3\cos 2(x + 35^\circ) + 2$	
	So f moves	

	∴ 35° horizontally to the left	
	and	
	∴ 2 units vertically upwards	
	to become g.	3

QUESTION 7

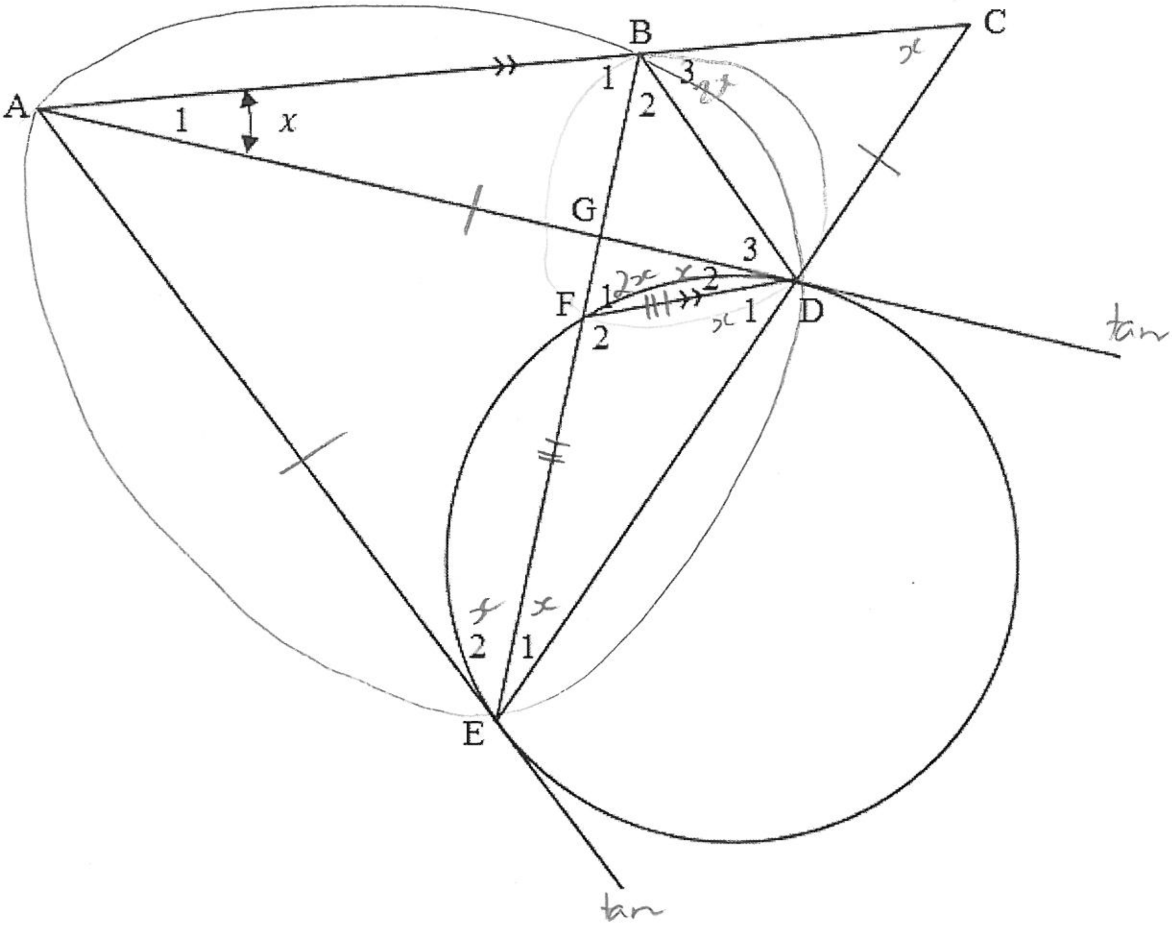
7.



	$\hat{B} = 90° + \theta$ ✓ ✓ S R opp ∠'s cyclic quad $= 180°$	
	$AC^2 = (3x)^2 + (2x)^2 - 2(3x)(2x)\cos(90° + \theta)$	✓
	$= 9x^2 + 4x^2 - 12x^2(-\sin\theta)$ ✓	
	$= 13x^2 + 12x^2\sin\theta$ ✓	
	$= x^2(13 + 12\sin\theta)$ ✓	
	$AC = \sqrt{x^2(13 + 12\sin\theta)}$	
	$= \sqrt{x^2}\sqrt{13 + 12\sin\theta}$	
	$= x\sqrt{13 + 12\sin\theta}$	6

QUESTION 8

8.

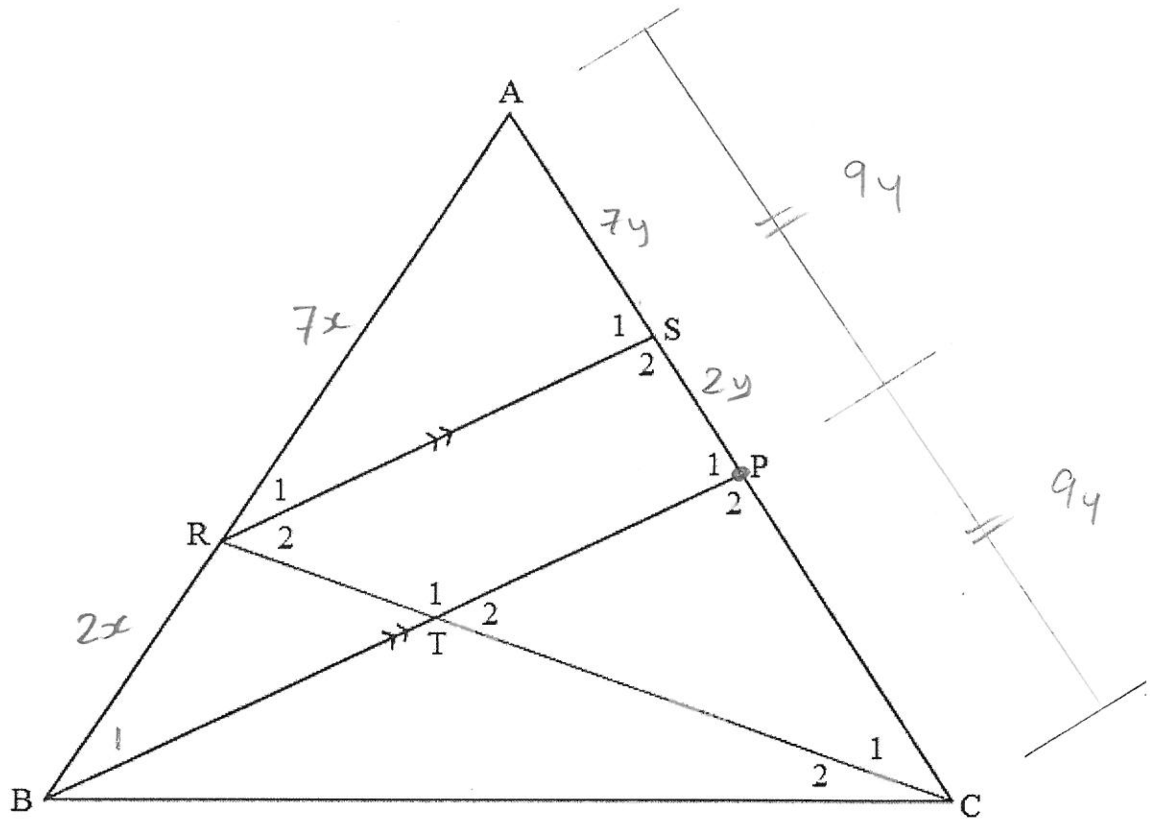


8.1.1.	$\hat{A}_1 = x$	
	$\hat{D}_2 = x$ ✓ SR alt ∠'s =, AC ∥ FD	
	$\hat{E}_1 = x$ ✓ S ✓ R ∠ tan chord	
	$\therefore \hat{A}_1 = \hat{E}_1$ both $= x$	3
8.1.2.	$\hat{A}_1 = \hat{E}_1$ (8.1.1)	
	∴ ABDE is a cyclic conv ∠'s in same	
	quad ⊙ segm = ✓R	1

8.2.	$\hat{D}_1 = x$ ✓SR	∠'s opp = sides	
	$\hat{C} = x$ ✓SR	corr ∠'s =, AC ∥ FD	
	$\therefore \hat{C} = \hat{A}_1$	both $= x$	2
8.3.	CD = AD ✓S	sides opp = ∠'s	
	AD = AE ✓S ✓R	tan's from ext	
		common pt =	
	$\therefore$ AE = CD	both = AE	3
8.4.	$\hat{F}_1 = 2x$ ✓SR	ext ∠ Δ	
	$\hat{D}_1 + \hat{D}_2 = \hat{E}_1 + \hat{E}_2$	∠'s opp = sides	
	$x + x =$		
	$2x = \hat{E}_1 + \hat{E}_2$ ✓SR		
	$\therefore \hat{B}_3 = 2x$ ✓S ✓R	ext ∠ cyclic quad	
	$\therefore \hat{B}_3 = \hat{F}_1$	both $= 2x$	
	$\therefore$ ABC is a tan	conv ∠ tan chord	
	to ⊙ B, F and D	✓R	5

QUESTION 9

9.

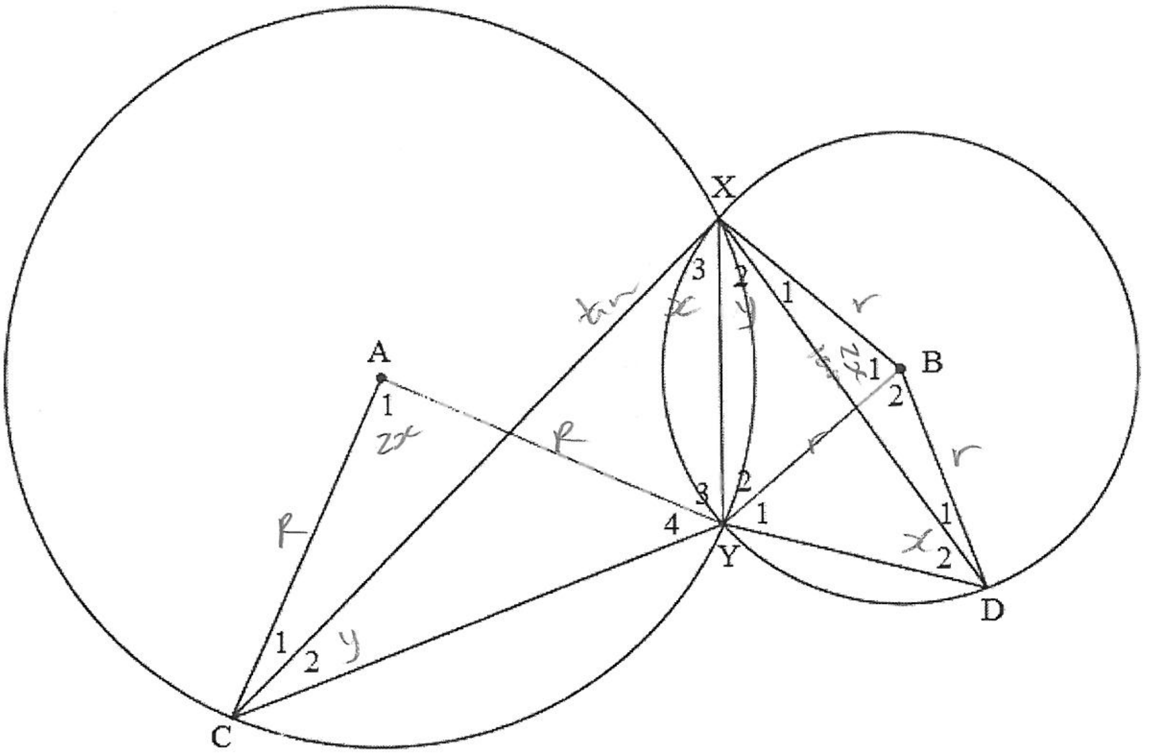


9.1.	$\frac{AR}{AB} = \frac{7}{9}$ $AR = 7x,\ AB = 9x\ \therefore RB = 2x$	
	$\frac{AS}{SP} = \frac{AR}{RB}$ ✓R line ∥ 1 side of Δ	
	$= \frac{7x}{2x}$	
	$= \frac{7}{2}$ ✓S $AS = 7y,\ SP = 2y\ \therefore PC = 9y$	
	$\therefore \frac{AS}{SC} = \frac{7y}{11y}$	
	$= \frac{7}{11}$ ✓S	3

9.2.	In Δ's AS_1R_1, AP_1B_1	
	1. $\hat{A} = \hat{A}$ ✓SR Common	
	2. $\hat{S}_1 = \hat{P}_1$ ✓SR corr $\angle$'s $=$, $RS \parallel BP$	
	$\therefore \Delta ASR \,\|\|\|\, \Delta APB$ ✓S AAA ✓R	
	$\frac{RS}{BP} = \frac{AR}{AB}$ $\Delta ASR \,\|\|\|\, \Delta APB$	
	$= \frac{7x}{9x}$	
	$= \frac{7}{9}$ ✓S	5
	✓ method	
9.3.	$\frac{\text{area } \Delta SAR}{\text{area } \Delta ABC} = \frac{\frac{1}{2}(7x)(7y)\sin\hat{A}}{\frac{1}{2}(9x)(18y)\sin\hat{A}}$	
	$= \frac{49}{162}$ ✓	2

QUESTION 10

10.



10.1.	Planning $XY.XY = DY.YC$	
	$\frac{XY}{DY} = \frac{YC}{XY}$	
	$\frac{XY}{DY} = \frac{YC}{YX}$ XYC, DYX	
	Answer	
	In Δ's $X_3Y_{3+4}C_2$, $D_2Y_{1+2}X_2$	
	Let $\hat{X}_3 = x$ and $\hat{X}_2 = y$	
	1. $\hat{X}_3 = \hat{D}_2$ ✓SR $\angle$ tan chord	
	2. $\hat{C}_2 = \hat{X}_2$ ✓SR $\angle$ tanchord	
	$\therefore \Delta XYC \parallel\!| \Delta DYX$ ✓S AAA ✓R	
	$\frac{XY}{DY} = \frac{YC}{YX}$ ✓S $\Delta XYC \parallel\!| \Delta DYX$	

	$\therefore XY^2 = DY \cdot YC$	5
10.2.	$\hat{A}_1 = 2x$ ✓S ✓R $\wedge$ @ centre = 2 $\wedge$ @	
	circumf	
	$\hat{B}_1 = 2x$ ✓SR $\wedge$ @ centre = 2 $\wedge$ @	
	circumf	
	$\therefore \hat{A}_1 = \hat{B}_1$ both $= 2x$	3
10.3.	In Δ's $C_{1+2}\,A_1\,Y_4$, $Y_2\,B_1\,X_{1+2}$	
	1. $\hat{A}_1 = \hat{B}_1$ ✓S (10.2.)	
	2. $\hat{C}_1 + \hat{C}_2 = \hat{Y}_4$ ✓SR radii, $\wedge$'s opp = sides	
	$\therefore \hat{Y}_4 = \frac{180° - 2x}{2}$ sum $\wedge$'s in $\Delta = 180°$	
	$= 90° - x$ ✓SR	
	Similarly, $\hat{X}_1 + \hat{X}_2 = 90° - x$ ✓S	
	$\therefore \hat{Y}_4 = \hat{X}_1 + \hat{X}_2$ both $= 90° - x$	
	$\therefore \Delta\, CAY \,\|\|\|\, \Delta\, YBX$ AAA ✓R	5

10.4. $\frac{YB}{CA} = \frac{YX}{CY}$ $\quad \triangle CAY \parallel\parallel \triangle YBX$

$\frac{r}{R} = \frac{YX}{CY}$ ✓ S

$(\ )^2$ bs :

$\frac{r^2}{R^2} = \frac{YX^2}{CY^2}$ ✓ num, method

$= \frac{DY.YC}{CY.CY}$ ✓ num (10.1.)

$= \frac{DY}{CY}$

3